Part 66 Cat. B1 / B2 Module 1

MATHEMATICS

Vilnius-2017

Converting Decimals to Common Fractions

To change a decimal fraction to a common fraction, count the number of digits to the right of the decimal point. Express the number as the numerator of a fraction whose denominator is *1* followed by the number of zeros that will equal the number of digits to the right of the decimal point.

Many times a dimension appearing in a maintenance manual or on a blueprint is expressed in decimal fractions. In order to use the dimension, it must be converted to some equivalent approximation applicable to the available measuring device. From the mechanic's standpoint, the steel rule will be the device most frequently used.

To change a decimal to the nearest equivalent fraction having a desired denominator, multiply the decimal by the desired denominator. The result will be the numerator of the desired fraction.

Example 1.1-20: When accurate holes of uniform diameter are required, they are first drilled $^{1}/_{64}$ inch undersize and reamed to the desired diameter. What size drill would be used before reaming a hole to *.763*?

1st step: The hole diameter *.763* must be converted to fraction. In order to convert multiply the decimal by the desired denominator of *64* and, for maintaining the same number, divide by 64, than calculate the nominator and round to a whole number:

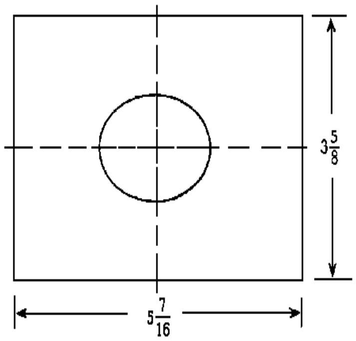
$$\frac{.763 \times 64}{64} = \frac{48.832}{64} = \frac{49}{64}\ inch.$$

2nd step: To determine the drill size, subtract $^{1}/_{64}$ inch from the finished hole size that gives $^{3}/_{4}$ in.

Converting Common Fractions to Decimals

To convert a common fraction, whether proper or improper, to a decimal, divide the numerator by the denominator. Add zeros to the right to permit carrying the quotient to the desired accuracy.

Example 1.1-21: The hole Ø 1.25 inch is drilled in a center of a sheet metal plate with dimensions as showed**.**



Find the distance, the center of the hole and its edges are from the plate edges and express the final results to the nearest 32^{nd}.

Solving this problem we start with changing mixed numbers to improper fractions and then to decimal expressions:

$$5\frac{7}{16} = \frac{87}{16} = 5.4375$$

$$3\frac{5}{8} = \frac{29}{8} = 3.625$$

Divide the decimal expressions by *2* to find the center of the plate (same the distance of the center from the edges of the plate):

$$\frac{5.4375}{2} = 2.7188$$

$$\frac{3.625}{2} = 1.813$$

To obtain the distance the hole edges are from the plate ones, we need find radius of the hole and subtract it from previous results:

$$2.7125 - \frac{1.25}{2} = 2.7125 - 0.625 = 2.0938\ (inch)$$

$$1.813 - \frac{1.25}{2} = 1.813 - 0.625 = 1.188\ (inch)$$

Finishing solution we express the final results to the nearest 32^{nd}. This will give us results which are measurable in the custom measurement system. To do this we need determine how many times contains 32 as divider

$$\frac{10\ 000}{32} = \frac{1250}{4} = \frac{625}{2}$$

And divide two desired nominators by previous result:

$$\frac{7\ 188}{625/_2} = 23.00 \text{ and } \frac{8\ 130}{625/_2} = 26.02,\ \frac{938}{625/_2} = 3.00 \text{ and } \frac{1880}{625/_2} = 6.01$$

The final result is finding mixing together the integer and fractional parts:

The hole center is	The hole edges are
$2\frac{23}{32}$ and $1\frac{26}{32}$ inches	$2\frac{323}{32}$ and $1\frac{6}{32}$ inches
from the edges of the plate	from the edges of the plate

Average Value

An average is a single value that is meant to typify a list of values. If all the numbers in the list are the same, then this number should be used. If the numbers are not all the same, an easy way to get a representative value from a list is to randomly pick any number from the list. However, the word *"average"* is usually reserved for more sophisticated methods that are generally found to be more useful.

1.2B ALGEBRA

Linear Equations and Their Solutions

A linear equation is an equation involving only the sum of constants or products of constants and the first power of a variable. Such an equation is equivalent to equating a first-degree polynomial to zero. These equations are called "*linear*" because they represent straight lines in Cartesian coordinates. A common form of a linear equation in two variables is

$$y = ax + b$$

where:

a is the slope of the line,

b is the y-intercept of the line,

x is the independent variable of the function y.

Examples of linear equations in two variables:

$$3x + 2y = 10$$

or

$$3a + 47b = 10b + 35$$

or

$$3x + y - 5 = 7x + 4y + 3.$$

This equation is solved as y dependence from different x values. Graphical way is the most visible. The common way for solving linear equations graphically is the use of *Cartesian coordinate system*.

Cartesian coordinate system

In mathematics, the Cartesian coordinate system is used to determine each point uniquely in a plane through two numbers, usually called the x-coordinate and the y-coordinate of the point. To define the coordinates, two perpendicular directed lines (the x-axis or abscissa and the y-axis or ordinate), are specified, as well as the unit length, which is marked off on the two axes (**Fig. 1.2-1**).

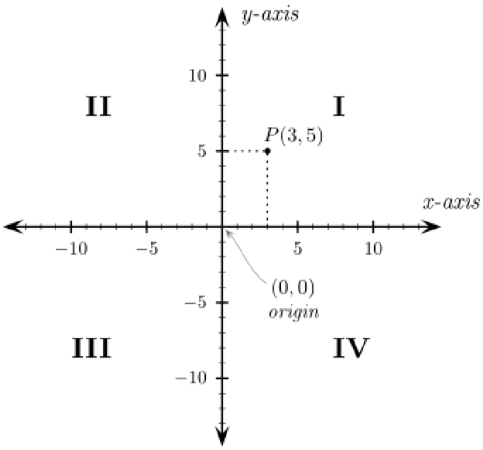


Figure 1.2-1. Cartesian Coordinate System

Cartesian coordinate systems are also used in space (where three coordinates are used) and in higher dimensions.

The modern Cartesian coordinate system in two dimensions (also called a rectangular coordinate system) is commonly defined by two axes, at right angles to each other, forming a plane (an *xy*-plane). The horizontal axis is normally labeled *x*, and the vertical axis is normally labeled *y*. The axes are commonly defined as mutually orthogonal to each other (each at a right angle to the other). The point of intersection, where the axes meet, is called the "*Origin*" normally labeled "*O*" (or "0").

To specify a particular point on a two-dimensional coordinate system, the *x* unit (*abscissa*) is indicated first, followed by the *y* unit (*ordinate*) in the form (*x*, *y*), an ordered pair.

The intersection of the two axes creates four regions, called quadrants (**Fig. 1.2-1**), indicated by the Roman numerals I, II, III, and IV. Conventionally, the quadrants are labeled counter-clockwise starting from the upper right ("*northeast*") quadrant. In the first quadrant, both coordinates are positive, in the second quadrant, *x*-coordinates are negative and *y*-coordinates positive, in the third quadrant both coordinates are negative and in the fourth quadrant, *x*-coordinates are positive and *y*-coordinates negative summarized below:

Quadrant	*x*-values	*y*-values
I	> 0	> 0
II	< 0	> 0
III	< 0	< 0
IV	> 0	< 0

Thus, any linear equation could be represented as shown at **Fig. 1.2-2.**

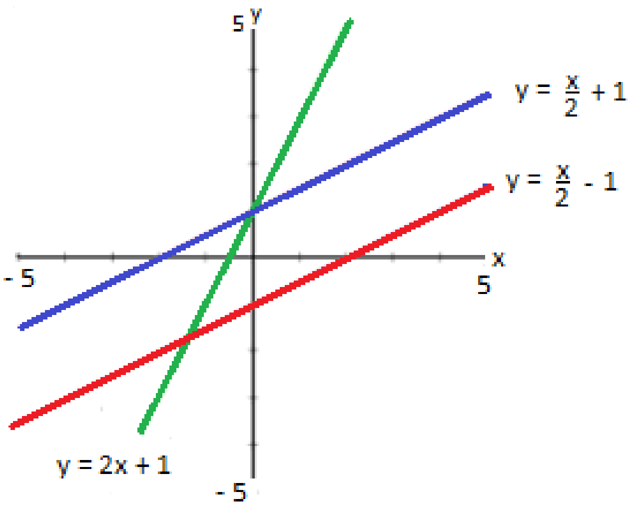


Figure. 1.2-2. Three lines represent different linear equations. The red and violet are of the same slope

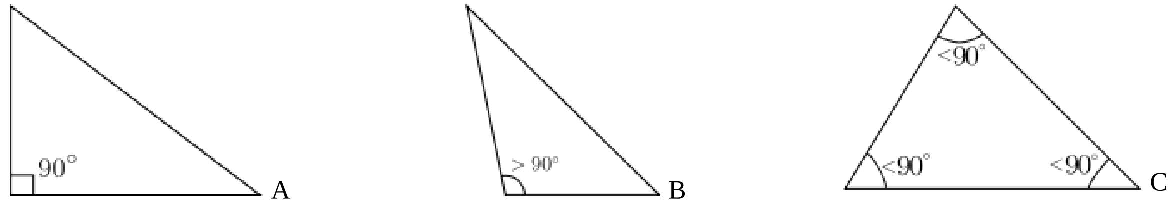


Figure 1.3-4. (A) Right triangle has one angle equal to 90°; (B) Obtuse triangle has one internal angle larger than 90° and (C) Acute triangle has internal angles all smaller than 90°

The altitude or height of a triangle is the perpendicular line drawn from the vertex to the base. In some triangles, as in **Fig. 1.3-5B**, it may be necessary to extend the base so that the altitude will meet it. The base of a triangle is the side upon which the triangle is supposed to stand. The term "base" denotes any side, and "height" denotes the length of a perpendicular from the point opposite the side onto the side itself.

Figure 1.3-5. (A) In this triangle the base is it longest side; (B) In obtuse triangle the base is extended in order the altitude will meet it

The area of any triangle may be calculated by using the formula:

$$A = \frac{1}{2}(b \times h),$$

where:

h - is the altitude of the triangle and

b (or other side) is the base.

Right Triangle Calculations

The special case is triangle's side calculation. The simplest type of triangles to calculate is the right triangle which follows Pythagorean Theorem.

In mathematics, the ***Pythagorean Theorem*** (American English) or ***Pythagoras' theorem*** (British English) is a relation among the three sides of a right triangle.

The theorem is as follows (**Fig. 1.3-6**):

In any right triangle, the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares whose sides are the two legs (the two sides that meet at a right angle).

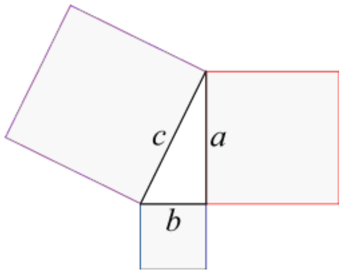


Figure 1.3-6. The sum of two squares whose sides are the two legs (*a* and *b*) is equal to the area of the square whose side is the hypotenuse (*c*)

This is usually summarized as follows:

The square of the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides:

$$c^2 = a^2 + b^2$$

and can be easily proven.

Lets make a square with a side $a + b$ in two different ways (**Fig. 1.3-7**).

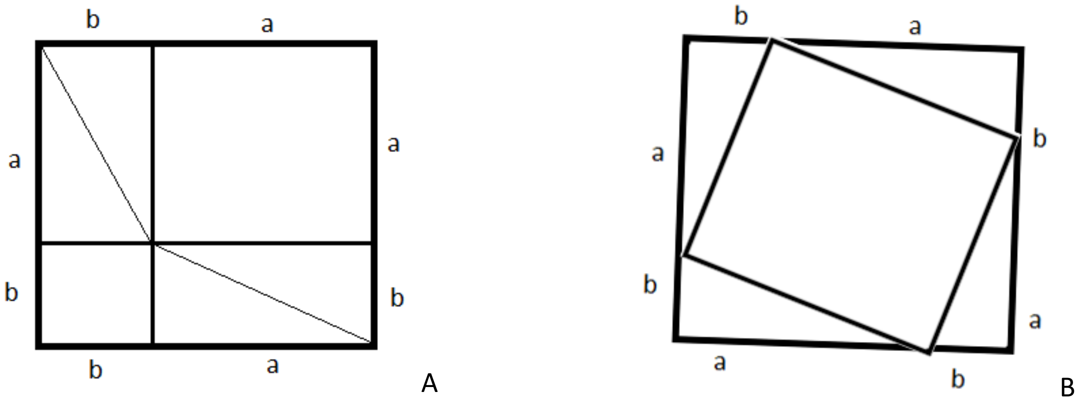


Figure 1.3-7. Pythagorean theorem prove

Both squares have their sites as $a + b$, so their areas are equal. The areas are composed of:

(A): $A = b^2 + a^2 + 2 \times a \times b;$

(B): $A = c^2 + 2 \times a \times b;$

If we subtract the second equation from the first one:

$$0 = b^2 + a^2 - c^2 \; or \; c^2 = b^2 + a^2.$$

And another obvious prove concerning the area or a triangle. Each small rectangle is composed of two triangles and has its area calculated as $A = a \times b$ with each triangle definitely being half of that. The same is valid for triangle of any shape, as in the case above one of the sites can be treated as triangle height.

$$y = \frac{a}{b}x - \frac{c}{b} = mx + n.$$

Using the last form common to the form of linear equation with one variable *x*, we can derive some useful properties plotting the possible graphical forms **(Fig. 1.3-19)** when the values of *m* and *n* are known.

On **Fig. 1.3-19** we have linear equations $y = 2x + 3$, $y = 5x - 3$ and $y = -2x + 3$. The second one differs from the first by coefficient at *x* (slope) and the free member (*y*-intersect), and the third one - by the sign before the variable (negative slope).

As we see, the larger coefficient at *x* in the second equation gives larger slope. More over, in the case of the first and the second equations the values of *y* rises simultaneously with the value of *x*, that means the function is "rising". In the last case (the third equation or function), the simple change of a sign from "+" to "-" caused a complete change in form - from rising to descending.

Analyzing the influence of the free member, we see the change in the point of intersection with 0y-axis: the larger the free member, the higher the intersection point is on the 0y-axis.

This allows formulating some rules for linear functions of two variables:

- The function $y = a_1x + b$ comparing to the function $y = a_2x + b$, has the larger slope to 0*x*-axis if $|a_1| > |a_2|$.
- the function $y = a_1x + b_1$ comparing to the function $y = a_2x + b_2$ has the higher point of intersect with 0y-axis if $b_1 > b_2$;
- When the coefficient at independent variable. of the .linear function $y = ax + b$ is $a > 0$, the function is rising;
- When the coefficient at independent variable of the linear function $y = ax + b$ is $a < 0$, the function is descending.

Properties of the Special Forms of the Linear Equation

There are some special forms of linear equations of two variables $y = ax + b$ (linear function) which are of particular interest and depend on which of members (y, a orb} are equal to 0 (zero) or strictly missing.

Lines Parallel to the Axis

If in a linear equation the *y* term is missing, as in

$$2x - 14 = 0$$

the equation represents a line parallel to the *Y*-axis and *7* units from it (**Fig. 1.3-20**). Similarly, an equation such as

$$4y - 8 = 0$$

which has no *x* term, represents a line parallel to the *X*-axis and *2 units* from it (**Fig. 1.3-20**).

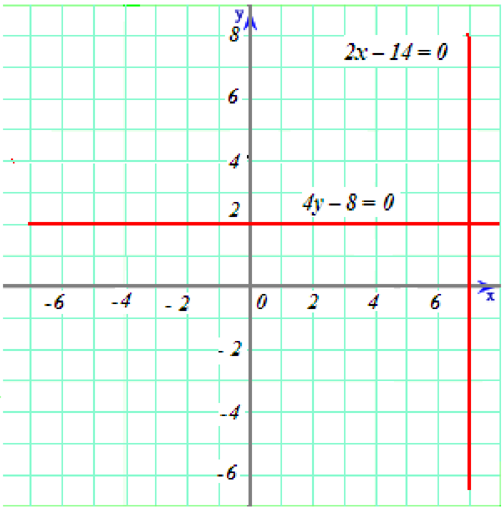


Figure 1.3-20. A linear equation of the type $ax + b = 0$

The fact that one of the two variables does not appear in an equation means that there are no limitations on the values the missing variable can assume. When a variable does not appear, it can assume any value from zero to plus or minus infinity. This can happen only if the line represented by the equation lies parallel to the axis of the missing variable.

Lines Passing Through the Origin

A linear equation, such as

$$4x + 3y = 0$$

has no constant term, represents a line passing through the origin.

This is since

$$x = 0, y = 0$$

satisfies any equation not having a constant term (**Fig. 1.3-21**).

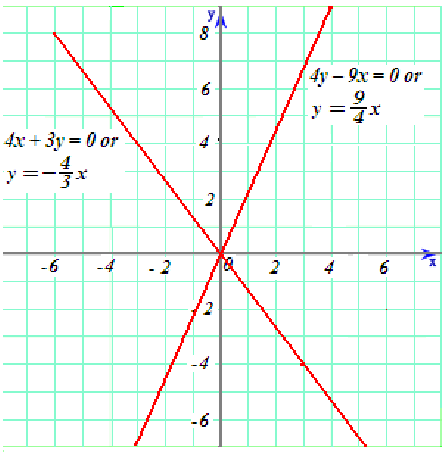


Figure 1.3-21. A linear equations of the type $ax + by = 0$